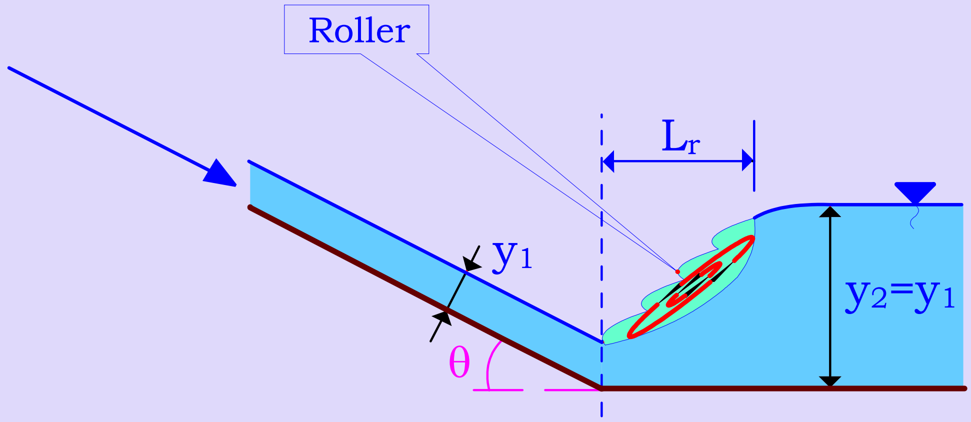
31. Hydraulic Jumps in Sloping Channels

Hydraulic jumps can occur in channels with larger bed slope that the gravitational forces acting on the flow must be included. The major problem in obtanining a useful solution to this problem are: (1) The term W sin θ is not well defined, because the length and shape of the jump are not well defined, (2) the specific weight of the fluid in the control volume changes significantly owing to air entrainment, and (3) the pressure terms cannot be accurately quantified.

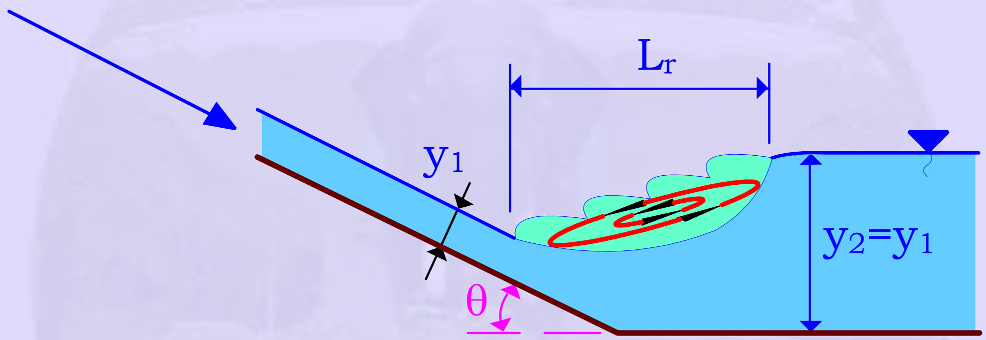
The earliest experiments carried out by Bidone on the hydraulic jump were actually performed in a sloping channel. Bazin in 1865 and Beebe and Riegel in 1917 also attempted to solve this problem. In 1927 Ellms attempted a theoretical and experimental study on sloping channel jumps (9^o to 17^o jumps). Yarnell in 1934 initiated an extensive study (600 tests) of the hydraulic jump in sloping channels (1:6, 1:3, 1:2, 1:1) which was not completed because of his untimely death in 1937. Rindlaub in 1935 conducted experimental investigation of 8.2^o, 12.5^o, 24.2^o and 30^o slopes with horizontal and most experiments were on 12.5^o slope . Bakhmeteff and Matzke conducted tests on 1:14 slope in 1936. Kindsvater(1944), using the unpublished Yarnell data, was the first investigator to develop a rational solution to the problem. Kindsvater also conducted studies on 1:3, 1:6 slope channel. Hickox conducted experiments in 1944 in channel with 1:3 slope. Dutta further studied in 1949 on slopes 1:3 , 1:4, 1:6. USBR in 1954 conducted extensive studies and slopes varied from 1 on 19 to 1 on 3.6 (3.01^o to 15.52^o) .Extensive studies have also been conducted by Bradley and Peterka (1957) and Argyropoulos (1962) and Rajarathnam (1966).

In discussing the equations and relationships available for hydraulic jumps in sloping channels, it is convenient to consider a number of cases. Jumps or sloping channel are classified into six types viz A, B, C, D, E and F. It is to be noted that, the end of the jump is taken at the end of the surface roller, unlike sequent depth in NHJ. Fig.1 shows these six formations.



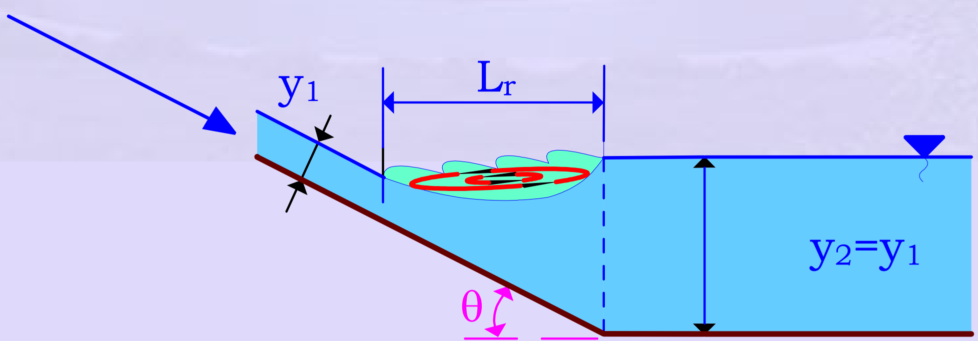
Type A

The jump occurs at the beginning of the horizontal bed



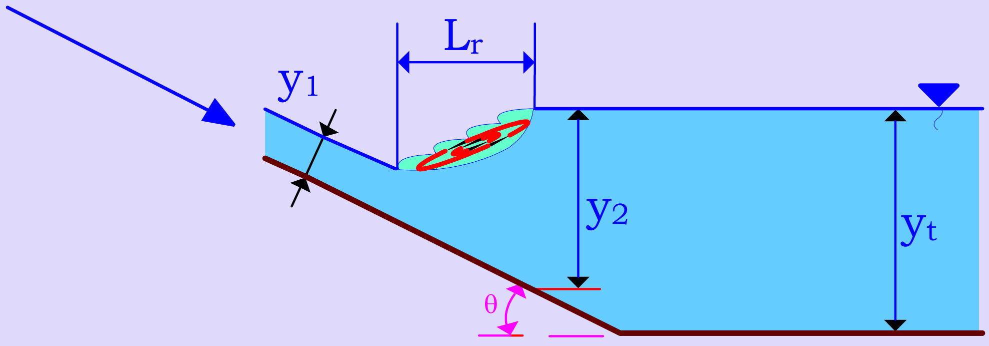
Type B

The jump overlaps the junction



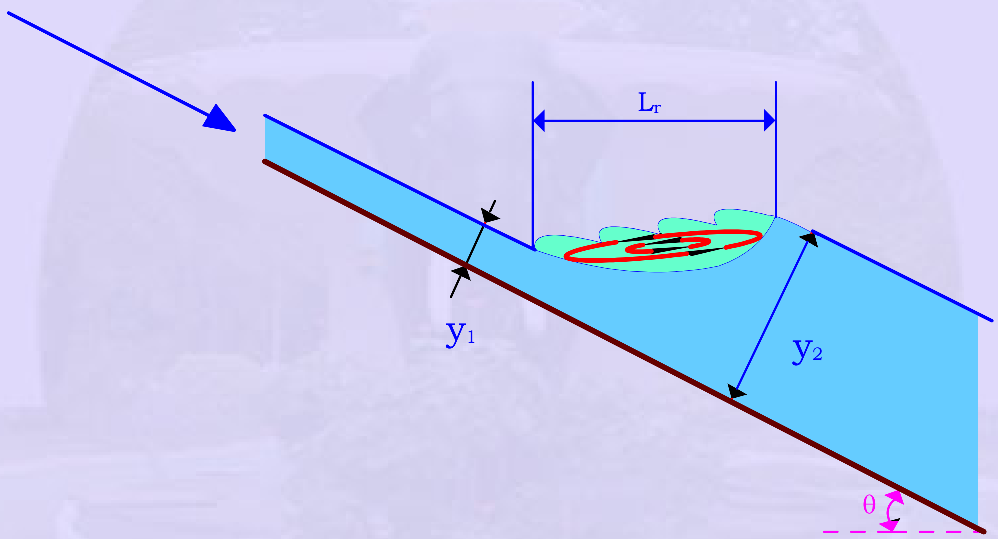
Type C

The jump ends exactly at the junction



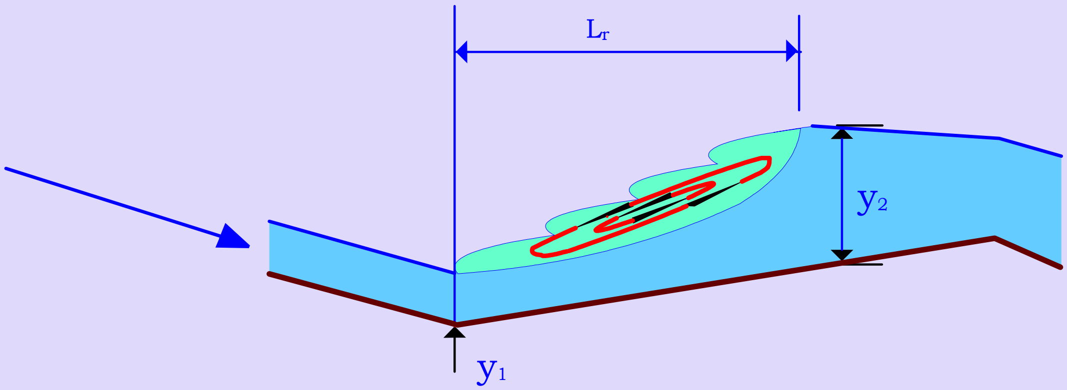
Type D

The jump occurs on the sloping channel



Type E

The jump is on the sloping channel without any break in the slope



Type F Adverse slope
The jump occurs in the adverse slope
Example: Near the downstream end
of the Draft Tube in Hydro Power Stations

Definition Sketch for types of Hydraulic Jumps on Sloping Channels

The notation are y_t is the tailwater depth, L_{rj} is length of the jump measured horizontally upto the end of the roller, y_1 is the supercritical depth of flow on the slope which is taken as constant, y_2 is the subcritical sequent depth corresponding to y_1, and y^*_2 is the subcritical depth for NHJ on horizontal flow.

In general, the end of the jump, in horizontal channels, is the section where the stream in the down stream attains the maximal steady elevation. This definition cannot be applied to the slopping channels. Because, even after the jump action is over, the water surface might be still rising owing to the flow expansion caused by the sloping bed. Kindsvater suggested that the end of the roller may be taken as the end of the jump. Thus $L_j = L_{rj}$. Hickox found that for slopes greater than 1:6 the end of the roller is practically as the section of maximal surface elevation. Further, the approach flow on the slope is assumed to be constant.

If the jump begins at the end of the sloping section, and occurs on level flood then $y_2^* = y_t$ and a type A jump, occurs which is similar to classical jump. If the end of the jump coincides with the intersection (junction) of the sloping and horizontal bed, a type C jump occurs.

If yt is less than that required for a type C jump but greater than y_2^*, the toe of the jump is on the slope and ends on the horizontal bed. This situation is termed a type B jump.

If y_t is greater than that required for a type C jump, then a type D jump occurs completely on the sloping section.

Type E jumps occurs on sloping beds which have no break in slope, and type F jump occurs only when there is adverse slopes such as in the case of tail race of the draft tube (could be submerged).

Types A to D are the most common jumps.

Kindsvater (1944) developed an equation for the type C jump for sequent depth.

$$\frac{y_2}{y_1'} = \frac{1}{2\cos\theta}\left[\sqrt{1+8F_1^2\left(\frac{\cos^3\theta}{1-2N_F\tan\theta}\right)}-1\right]$$

in which N_F is an empirical factor related to the length of the jump and θ is the angle of the bed slope. N_F depends on the slope angle. The above equation can be written as

$$\frac{y_2}{y_1'} = \frac{1}{2}\left(\sqrt{1+8G_1^2}-1\right)$$

in which

$$G_1^2 = \Gamma_1^2\, F_1^2 \quad \text{and}$$

$$\Gamma_1^2 = \frac{\cos^3\theta}{1-2\,N_F\tan\theta}$$

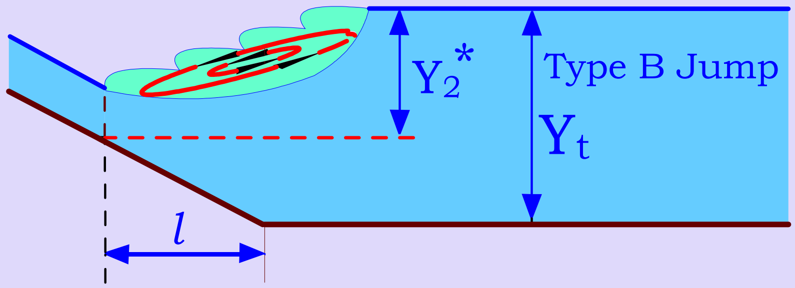
$$y_1' = y_1 / \cos\theta$$

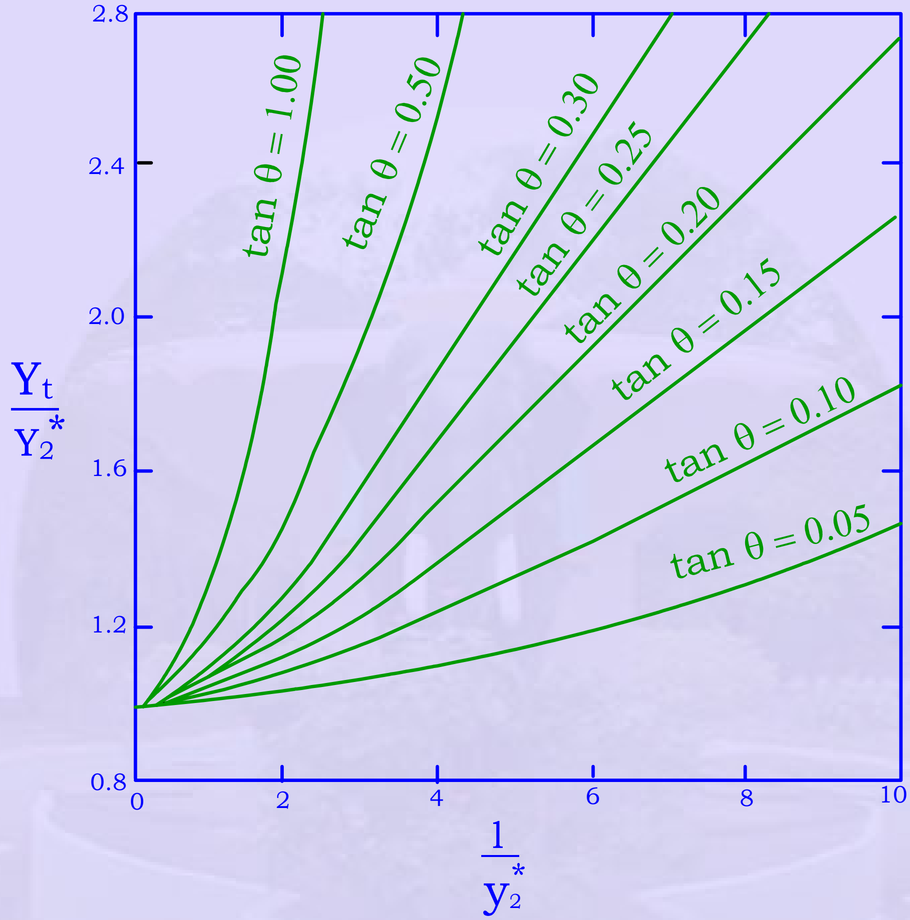
Bradley and Peterka (1957 a) and Peterka (1963) found that N_F depends mainly upon θ, and Rajaratnam (1967) provided the following relationships.

$$\Gamma_1 = 10^{0.027\theta} \quad \text{or} \quad \log\Gamma_1 = 0.027\,\theta$$ where θ is in degrees.

Bradley and Peterka (1957a) and Peterka (1963) also found that above equation could also be applied to the Type D jump, only with the jump that $y_2 \neq y_t$. Regarding B type a graph was presented. The Figure shows that in addition to the curve presented by them two curves of $\tan\theta = 0.05$ and 1.0 are added. The curves $\frac{y_2}{y_2^*}$ are plotted against $\frac{l}{y_2^*}$

in which l is the distance of the toe of the jump from the junction. Even though Analytic solution for the B jump has not yet been developed, Bradley and Peterka (1957) and Peterka (1963) have developed a graphical solution for this type of jump based on laboratory investigations.



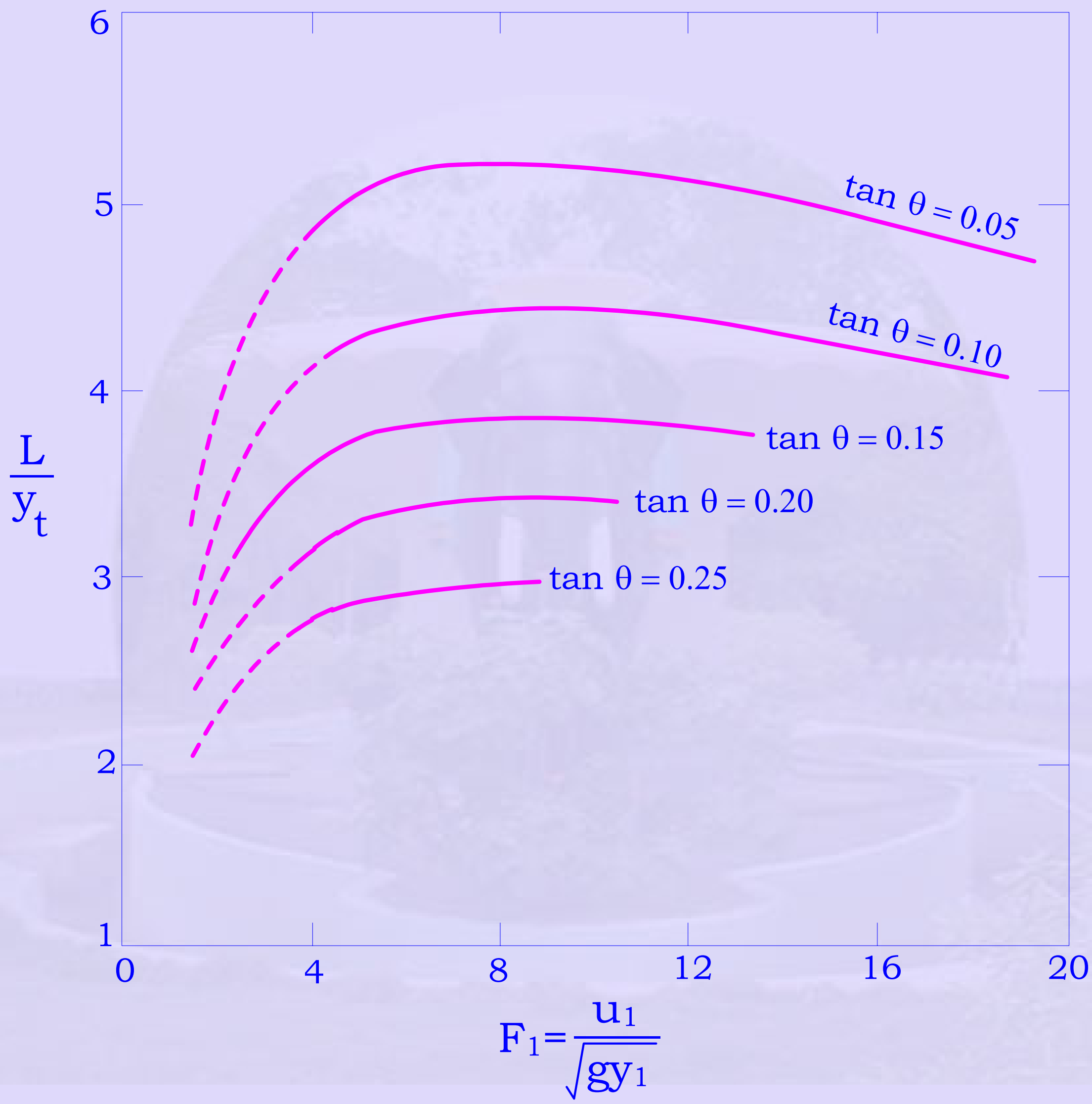


Solution for Type B jump

Peterka A.J. "Hydraulic design of Stilling basins and Energy Dissipators", Engineering Monograph, Number - 25, U.S. Bureau of Reclamation, Denver 1963.

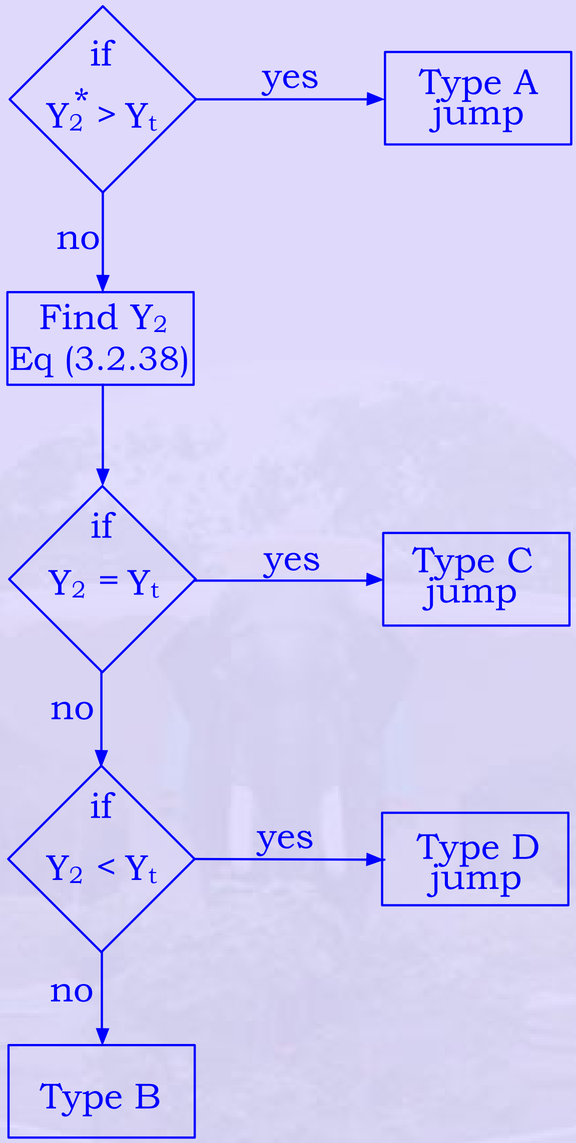
Rajarathnam N. "Hydraulic Jumps", Advances in Hydro Science, Volume - 4, Academic Press, New York, 1967, pp 197 - 280

The first step in classifying the jump (a) the slope of channel must be considered along with the initial depth of super critical flow and the tail water conditions. Peterka assumed the initial depth as constant on the sloping bed and developed a procedure to classify the jump on sloping bed. The results presented by Bradley and Peterka regarding the length of the D - jump are shown in figure. This figure may also be used for estimating the lengths of the Type - B and Type - C jumps. The energy loss for the Type - A jump mat be estimated from the standard equation for Normal Hydraulic Jump (NHJ). For Jumps Types C and D, expressions can be derived. The reader is referred to the book by Richard H. French "Open Channel Hydraulics", Mc GrawHill Company, 1986.



Hydraulic jump length in sloping channels for jump types B, C, and D. (Peterka, 1963)

French provided a flow chart for classifying the hydraulic jump type in sloping channels.



Determination of hydraulic jump type in sloping channels

Energy Loss in sloping channels for Types C and D

$$E_1 = L_{rj} \tan\theta + \frac{y_1}{\cos\theta} + \frac{\bar{v}_1^2}{2g}$$

in which L_{rj} is the length of the roller and $\bar{v}_1$ is the approach velocity of the super critical flow. The energy at the end of the jump is given by

$$E_2 = y_2 + \frac{\bar{v}_2^2}{2g}$$

in which $\bar{v}_2$ is the velocity at the downstream of jump. Then the relative energy loss can be expressed as

$$\frac{\Delta E}{E_1} = \frac{\left[1 - \frac{y_2}{y_1}\right] + \frac{F_1^2}{2}\left[1\left(\frac{1}{\left(\frac{y_2}{y_1}\right)^2}\right)\right] + \left(\frac{L_{rj}\ y_t}{y_t y_1}\right)\tan\theta}{1 + \left(\frac{F_1^2}{2}\right) + \left(\frac{L_{rj}\ y_t}{y_t y_1}\right)\tan\theta}$$

in which $y_1 \simeq \frac{y_1}{\cos\theta}$. In general, the above equation should not be used in situations when $F_1 < 4.0$ as $\Delta E / E_1$ is sensitive to the ratio $\frac{L_{rj}}{y_2}$.

Hydraulic jump

$$\frac{y_2}{y_1} = 0.5\left(\sqrt{8F_1^2 + 1} - 1\right)$$

$$\eta = 0.5\left(\sqrt{8F_1^2 - 1} - 1\right)$$

Also

$$2F_1^2 = \eta^2 + \eta$$

Van Driest modified the equation incorporating the correction factor

$$2F_1^2 = \frac{\eta\left[\eta^2 + \beta_2' - \beta_1' + \frac{R}{P_1}\right]}{\left(\alpha_1 + \alpha_{1,T}\right)\eta - \left(\alpha_2 + \alpha_{2,T}\right)} \qquad (1)$$

in which α is the coefficient of velocity correction, β' is the pressure correction factor, R / P_1 is the effect of friction, relative to the resulting static pressure P_1 in the approach flow. Friction factor can reduce the depth by 2 to 8 %.

If σ is the ratio of mass densities of $\frac{\rho_{\omega a}}{\rho_\omega} = \frac{\text{air water}}{\text{water}}$ then three different cases are possible.

(i) $\sigma_2 = \sigma_1 < 1.0$ same air concentration at the beginning and at the end of jumps.

(ii) $\sigma_2 = 1.0,\ \sigma_1 < 1.0$ air entrained approach flow and air free at the downstream end.

(iii) $\sigma_2 \neq \sigma_1 < 1.0$ unequal air concentration before and after the jump.

The respective equation for hydraulic jump can be written as follows.

Case (i)

$$2\eta F_1^2\sigma_{1,2} + \eta = 2F_1^2\sigma_{1,2} + \eta^3 \qquad (2)$$

$$\eta^3 - \eta\left[1 + 2F_1^2\sigma_{1,2}\right] + 2F_1^2\sigma_{1,2} = 0 \qquad (2a)$$

$$\eta = \left[\sqrt{\sigma_{1,2} 8F_1^2 + 1} - 1\right] \qquad (3)$$

Case (ii)

$$\eta^3 - \eta\left[\frac{1}{\sigma} + 2F_1^2\right] + 2F_1^2 = 0 \qquad (4)$$

$$2F_1^2 = \frac{\eta^3 - \frac{\eta}{\sigma}}{\eta - 1} \qquad (5)$$

Case (iii)

$$\eta^3 - \eta\left[\frac{\sigma_2}{\sigma_1} + 2F_1^2\sigma_2\right] + 2F_1^2\sigma_2 = 0 \qquad (6)$$

if $\sigma_1 = \sigma_2$ equation 6 reduces to eq.2

if $\sigma_2 = 1.0$ equation 6 reduces to 4 and 5

If $\sigma_1 = \sigma_2$ equation 6 reduces to equation 2.

If $\sigma_2 = 1.0$ equation 6 reduces to 4 and 5.

Further incorporating the air entrainment, except, the equation 1 can be expressed as

$$\frac{\eta^3\beta_2'}{\sigma_2} - \eta\left[\frac{\eta^3\beta_1'}{\sigma_1} + 2\left(\alpha_1 + \alpha_{1,T}\right)F_1^2 - \frac{R}{P_1}\right] + 2\left(\alpha_2 + \alpha_{2,T}\right)F_1^2 = 0$$

or

$$2F_1^2 = \frac{\eta\left(\dfrac{\eta^2\beta_2'}{\sigma_2} - \dfrac{\beta_1'}{\sigma_1} - \dfrac{R}{P_1}\right)}{\left(\alpha_1 + \alpha_{1,T}\right)\eta - \left(\alpha_2 + \alpha_{2,T}\right)}$$

Specific force equation to hydraulic jump considering air entrainment in the approach flow can be written as,

$$\rho_{wl} Q_{wl} \overline{V}_1 + \frac{1}{2}\rho_{wL} g b h_{1wL}^2 = \rho_{wL} Q_{wL} \overline{V}_2 + \frac{1}{2}\rho_{wl} g b h_{2wL}^2$$

writing $\rho_{wl} Q_{wL} \approx \rho_w Q_w$, and $\eta = \dfrac{h_{2w}}{h_{1w}}$ then

$$\rho_w Q_w \overline{V_1} + \frac{1}{2}\rho_w g \sigma_{1,2} \frac{b h_{1w}^2}{\sigma_{1,2}^2} = \frac{\rho_w Q_w \overline{V}_1}{\eta} + \frac{1}{2}\rho_w g \sigma_{1,2} \frac{b h_{2w}^2}{\sigma_{1,2}^2}$$

As $Q_w = V_1 b h_{1w}$ then

$$\frac{V_1^2}{g} b h_{1,w} + \frac{h_{1w}^2}{2\sigma_{1,2}} b = \frac{V_1^2}{g} b \frac{h_{1,w}}{\eta} + \frac{h_{2,w}^2}{2\sigma_{1,2}} b$$

When simplified

$$\frac{2\overline{V}_1^2}{g h_{1w}} \eta \sigma_{1,2} + \eta = 2\frac{\overline{V}_1^2}{g h_{1w}} \sigma_{1,2} + \eta^3$$

With $F_1 = \dfrac{V_1}{\sqrt{g h_{1,w}}}$ it simplifies to equation 2.

$$\eta^3 + 2F_1^2 \sigma_{1,2} = 2\eta F_1^2 \sigma_{1,2} + \eta$$

Rajarathnam's Equation for Pre entrained Jump

$$\eta'^{\,3} - \eta'\left[\sigma + 2F_1'^{\,2}\,\sigma C_1^2\right] + 2F_1'^{\,2}\sigma^2\,C_2^2 = 0$$

in which $\eta' = \dfrac{h_{2,w}}{h_{1wl}} = \sigma\eta$ $\quad F_1'^{\,2} = \dfrac{\overline{V}_1^2}{g h_{1,wl}} = \sigma F_1^2$

$\sigma = \sigma_1$ since $\sigma_2 = 1.0$, C_1 and C_2 are the correction factors for non uniform distribution over the depth of cross section considered. Rajarathnam suggested $C_1 = 1.12$ and $C_2 = 1.14$

Specific Force

$$\rho_{wl} Q_{wl} = \rho_w \sigma Q_w \frac{1}{\sigma} = \rho_w Q_w \qquad (1)$$

Hydrostatic Pressure

$$P_w = \frac{1}{2} \rho_w g b h_{wl}^2, \qquad h_{wl} = \frac{h_w}{\sigma}$$

With air entrainment

$$P_{wl} = \frac{1}{2} \rho_{wl}\ g b h_{wl}^2 = \frac{1}{2} \rho_w g b \frac{h_w^2}{\sigma} = \frac{P_w}{\sigma}$$

Specific Force of Water

$$S_w = \rho_w Q_w \overline{V}_w + \frac{1}{2} \rho_w g b h_w^2$$

$$S_{wl} = \rho_{wl} Q_{wl} \overline{V}_{wl} + \frac{1}{2} \rho_{wl}\ g b h_{wl}^2$$

$$= \rho_w Q_w \overline{V}_w + \frac{1}{2} \rho_w g b \frac{h_w^2}{\sigma}$$

$$\therefore\ S_{wl} - S_w = \frac{1}{2} \rho_w g b h_w^2 \left(\frac{1}{\sigma} - 1 \right)$$

$$\therefore\ S_{wl} > S_w$$

$\rho_{w,l}$ = Mass density of aerated water

ρ_w = Mass density of water

$$\sigma = \frac{\rho_{w,l}}{\rho_w}$$

h_{wl} = depth of self aerated water

h_w = depth of water

b = width of the channel

P = Hydrostatic pressure