9.2 One-way Slabs

This section covers the following topics.

- Introduction
- Analysis and Design

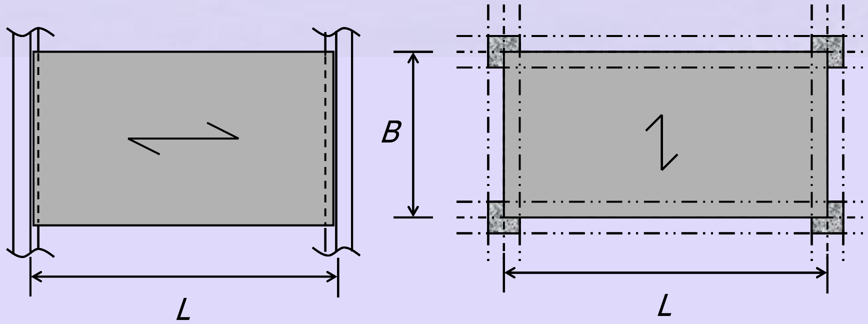
9.2.1 Introduction

Slabs are an important structural component where prestressing is applied. With increase in the demand for fast track, economical and efficient construction, prestressed slabs are becoming popular. The slabs are presented in two groups: **one-way slabs** and **two-way slabs**. The two-way slabs are presented in details in Sections 9.3 and 9.4.

Rectangular slabs can be divided into the two groups based on the support conditions and length-to-breadth ratios. The one-way slabs are identified as follows.

1) When a rectangular slab is supported only on two opposite edges, it is a one-way slab spanning in the direction perpendicular to the edges. Precast planks fall in this group.
2) When a rectangular slab is supported on all the four edges and the length-to-breadth (L / B) ratio is equal to or greater than two, the slab is considered to be a one-way slab. The slab spans predominantly in the direction parallel to the shorter edge.

The following sketches show the plans of the two cases of one-way slabs. The spanning direction in each case is shown by the double headed arrow.



(a) (b)

(a) Supported on two opposite edges (b) Supported on all edges (L/B > 2)

Figure 9-2.1 Plans of one-way slabs

A slab in a framed building can be a one-way slab depending upon its length-to-breadth ratio. A one-way slab is designed for the spanning direction only. For the transverse direction, a minimum amount of reinforcement is provided. A hollow core slab is also an example of a one-way slab. A ribbed floor (slab with joists) made of precast double tee sections, is analysed as a flanged section for one-way bending.

Other types of rectangular slabs and non-rectangular slabs are considered to be two-way slabs. If a rectangular slab is supported on all the four sides and the length-to-breadth ratio is less than two, then it is a two-way slab. If a slab is supported on three edges or two adjacent edges, then also it is a two-way slab. A slab in a framed building can be a two-way slab depending upon its length-to-breadth ratio. A two-way slab is designed for both the orthogonal directions.

A slab is prestressed for the following benefits.

1) Increased span-to-depth ratio

   Typical values of span-to-depth ratios in slabs are given below.

Non-prestressed slab	28:1
Prestressed slab	45:1

2) Reduction in self-weight
3) Section remains uncracked under service loads

   ⇒ Increased durability
4) Quick release of formwork

   ⇒ Fast construction
5) Reduction in fabrication of reinforcement
6) More flexibility in accommodating late design changes.

Precast planks are usually pre-tensioned. Cast-in-situ slabs are post-tensioned. Post-tensioned slabs are becoming popular in office and commercial buildings and parking structures, where large column-free spaces are desirable. The maximum length of a post-tensioned slab is limited to 30 to 40 m to minimise the losses due to elastic shortening and friction.

Slabs can be composite for the benefits of reduction in form work, cost and time of construction and quality control. A precast plank can be prestressed and placed in the final location. A topping slab is overlaid on the precast plank. The grades of concrete in the two portions can be different. The following sketches show the sections of some one-way slabs.

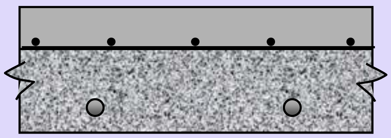


Figure 9-2.2 Cross-section of a composite slab

9.2.2 Analysis and Design

One-way slabs are analysed and designed for the spanning direction similar to rectangular beams. The analysis and design is carried out for the width of the plank or a unit width (say 1 m) of the slab. For continuous slabs, the moment coefficients of **IS:456 - 2000 (Table 12)** can be used.

The analysis and design procedures for simply supported rectangular beams are covered in Sections 3.2 to 3.6 and Sections 4.2 to 4.6, respectively. These materials are briefly reproduced here.

Preliminary Design

1) Select the material properties f_{ck} and f_{pk}. Here, f_{ck} is the characteristic compressive strength of concrete and f_{pk} is the characteristic tensile strength of prestressing steel.
2) Determine the total depth of slab (h), based on the span to effective depth ratio (L / d), given in **Clause 22.6** of **IS:1343 - 1980**. Consider $d \approx h - 25$ mm. Round off h to a multiple of 10 mm.
3) Calculate the self weight.
4) Calculate the total moment (M_T) including moment due to self weight (M_{sw}).
5) Estimate the lever arm (z).

   $z = 0.65\ h$ if M_{sw} is large (say $M_{sw} > 0.3\ M_T$)

   $z = 0.5\ h$ if M_{sw} is small.
6) Estimate the effective prestress (P_e).

   $P_e = M_T / z$ if M_{sw} is large

   $P_e = M_{IL} / z$ if M_{sw} is small.

   Here, the moment due to imposed loads is given as $M_{IL} = M_T - M_{sw}$.
7) Considering $f_{pe} = 0.7\ f_{pk}$, calculate area of prestressing steel $A_p = P_e / f_{pe}$.

8) Check the area of cross section (A) A = 1000 mm × h mm. The average stress C/A should not be too high as compared to 50% $f_{cc,all}$.

Final Design

The final design involves the checking of the stresses in concrete at transfer and under service loads with respect to the allowable stresses. The allowable stresses depend on the type of slab (Type 1, Type 2 or Type 3). Here, the steps of final design are explained for Type 1 slabs only. For Type 1 slabs, no tensile stress is allowed at transfer or under service loads.

For small moment due to self-weight ($M_{sw} \leq 0.3\, M_T$), the steps are as follows.

1) Calculate eccentricity (e) to locate the centroid of the prestressing steel (CGS).

The lowest permissible location of the compression (C) due to self-weight is at the bottom kern point (at a depth k_b below CGC) to avoid tensile stress at the top. The design procedure based on the extreme location of C gives an economical section. For this location of C, the following equation can be derived.

$$e = \frac{M_{sw}}{P_0} + k_b \quad \textbf{(9-2.1)}$$

The magnitude of C or T is equal to P_0, the prestress at transfer after initial losses.

The value of P_0 can be estimated as follows.

a) $P_0 = 0.9\, P_i$ for pre-tensioned slab

b) $P_0 = P_i$ for post-tensioned slab

Here, P_i is the initial applied prestress.

$$P_i = (0.8 f_{pk})\, A_p \quad \textbf{(9-2.2)}$$

The permissible prestress in the tendon is $0.8 f_{pk}$.

2) Re-compute the effective prestress P_e and the area of prestressing steel A_p.

For the extreme top location of C under service load, the shift of C due to the total moment gives an expression of P_e.

$$P_e = \frac{M_T}{e + k_T} \quad \textbf{(9-2.3)}$$

For solid rectangular slab, $k_b = k_t = h / 6$.

Considering $f_{pe} = 0.7f_{pk}$, the area of prestressing steel is recomputed as follows.

$$A_p = P_e / f_{pe} \quad \textbf{(9-2.4)}$$

The number of tendons and their spacing is determined based on A_p. The value of P_0 is updated.

3) Re-compute e with the updated values of A_p and P_0.
If the variation of e from the previous value is large, another cycle of computation of the prestressing variables can be undertaken.

For large M_{sw} if e violates the cover requirements, e is determined based on cover.

4) Check the compressive stresses in concrete
For the limiting no tension design at transfer, the stress at the bottom (f_b) is given as follows.

$$f_b = -\frac{P_0}{A}\frac{h}{c_t} = -\frac{2P_0}{A} \quad \textbf{(9-2.5)}$$

The stress should be less than $f_{cc,all}$, where $f_{cc,all}$ is the allowable compressive stress in concrete at transfer. The condition to satisfy can thus be written as $| f_b | \leq f_{cc,all}$.

For the limiting no tension design at service, the stress at the top (f_t) is given as follows.

$$f_t = -\frac{P_e}{A}\frac{h}{c_b} = -\frac{2P_e}{A} \quad \textbf{(9-2.6)}$$

The stress should be less than $f_{cc,all}$, where $f_{cc,all}$ is the allowable compressive stress in concrete at service. The condition to satisfy can be written as $| f_t | \leq f_{cc,all}$.

For Type 2 and Type 3 slabs, the tensile stress should be restricted to the allowable values. For a continuous slab, a suitable profile of the tendons is selected similar to that in continuous beams. The design of continuous beams is covered in Sections 8.2 and 8.3.

When the value of e is fixed (in either pre-tension or post-tension operations), the design steps are simpler. If the tendons are placed at the CGC (e = 0), then the uniform compressive stress due to prestress counteracts the tensile stress due to service loads. To have zero stress at the bottom under service conditions, the value of P_e can be directly calculated from the following equation.

$$\frac{P_e}{A} = \frac{M_T}{Z_b}$$

$$\text{or,} \quad P_e = A\frac{M_T}{Z_b} \qquad \textbf{(9-2.7)}$$

Z_b is the section modulus. The above expression is same as $P_e = M_T / k_t$, with $e = 0$. The stresses at transfer can be checked with an estimate of P_0 from P_e.

5) Checking for shear capacity

The shear is analogous to that generates in a beam due to flexure. The calculations can be for unit width of the slab. The critical section for checking the shear capacity is at a distance effective depth 'd' from the face of the beam, across the entire width of the slab. The critical section is transverse to the spanning direction. The shear demand (V_u) in the critical section generates from the gravity loads in the tributary area.

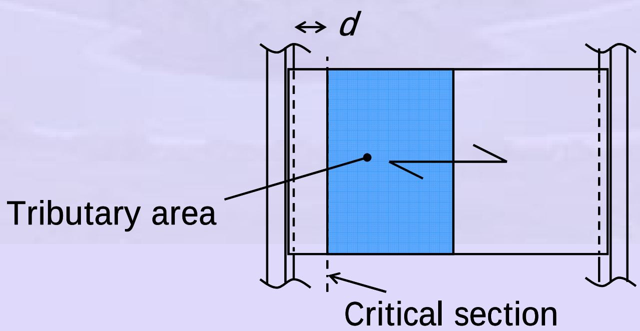


Figure 9-2.3 Tributary area and critical section for shear

For adequate shear capacity, $V_{uR} \geq V_u$ where, $V_{uR} = V_c$, the shear capacity of uncracked concrete of unit width of slab. The expression of V_c is given in Section 5.2, Design for Shear (Part I). If this is not satisfied, it is preferred to increase the depth of the slab to avoid shear reinforcement.

6) Provide transverse reinforcement based on temperature and shrinkage.

As per **IS:456 - 2000**, **Clause 26.5.2.1**, the minimum amount of transverse reinforcement ($A_{st,min}$ in mm^2) for unit width of slab is given as follows.

$$A_{st,min} = 0.15\%\ 1000\ h \quad \text{for Fe 250 grade of steel}$$
$$= 0.12\%\ 1000\ h \quad \text{for Fe 415 grade of steel.}$$

Usually the transverse reinforcement is provided by non-prestressed reinforcement. The minimum reinforcement is sufficient for the transverse moment due to Poisson's effect and small point loads. For a heavy point load, transverse reinforcement needs to be computed explicitly.

The following example shows the design of a simply supported precast prestressed composite slab.

Reference:

Santhakumar, A.R., *Partially Precast Composite PSC Slab*, Published by Building Technology Centre, Anna University, Chennai.

Example 9-2.1

Design a simply supported precast prestressed (Type 1) composite slab for the following data.

Width of the slab	**= 0.3 m**
Clear span	**= 2.9 m**
Effective span (*L*)	**= 3.1 m**
Thickness of the precast plank	**= 50 mm**
Thickness of the cast-in-situ topping slab	**= 50 mm**
Grade of concrete in precast plank	**: M60**
Grade of concrete in topping slab	**: M15**

The pre-tensioned tendons are located at mid depth of the precast slab. During the casting of the topping, planks are not propped.

Live load = 2.0 kN/m^2

Load due to floor finish = 1.5 kN/m^2.

Solution

1) Calculation of moments.

Load per unit area

Weight of precast plank	= 1.25 kN/m^2
Weight of topping slab	= 1.25 kN/m^2
Weight of floor finish	= 1.50 kN/m^2
Live load	= 2.00 kN/m^2
Total	= 6.00 kN/m^2

Total moment (M_T) along the width of the slab is given as follows.

$$\frac{wBL^2}{8} = \frac{6 \times 0.3 \times 3.1^2}{8}$$
$$= 2.16 \text{ kNm}$$

The individual moments are calculated based on the proportionality of the loads.

M_{SW} = moment due to self weight of precast plank

= 2.16 × (1.25 / 6.00) = 0.45 kNm

M_{top} = moment due to weight of topping slab

= 2.16 × (1.25 / 6.00) = 0.45 kNm

M_{fin} = moment due to weight of floor finish

= 2.16 × (1.50 / 6.00) = 0.54 kNm

M_{LL} = moment due to live load

= 2.16 × (2.00 / 6.00) = 0.72 kNm.

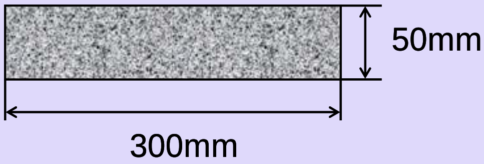
2) Calculation of geometric properties.

Precast section

Area

$A_1 = 300 \times 50 = 15{,}000 \text{ mm}^2$

Moment of inertia



$$I_1 = \frac{1}{12} \times 300 \times 50^3$$
$$= 3{,}125{,}000 \text{ mm}^4$$

Distance to the extreme fibres

$$c_b = c_t = \frac{50}{2}$$
$$= 25 \text{ mm}$$

Section moduli

$$Z_b = Z_t = \frac{3{,}125{,}000}{25}$$
$$= 125{,}000 \text{ mm}^3$$

Composite section

Since the grades of concrete are different for the precast- prestressed (PP) and cast-in-situ (CIS) portions, an equivalent (transformed) area is calculated. The CIS portion is assigned a reduced width based on the equivalent area factor (modular ratio).

Equivalent area factor = Modulus of CIS / Modulus of PP

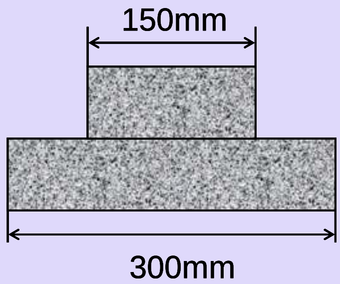
= √(Grade of CIS / Grade of PP)

=√15/60

= 0.5

Composite section

Equivalent section



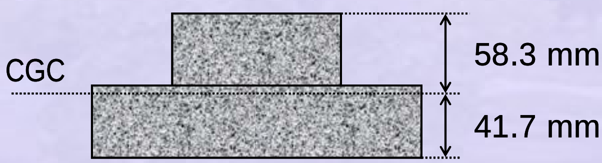
Location of CGC from bottom

A_{top} = 50 × 150 = 7,500 mm²

A_{bot} = 50 × 300 = 15,000 mm²

A_2 = A_{top} + A_{bot} = 22,500 mm²

$$\bar{y} = \frac{A_{top} \times 75 + A_{bot} \times 25}{A}$$
$$= \frac{937{,}500}{22{,}500}$$
$$= 41.7 \text{ mm}$$



Moment of inertia

$$I_{top} = \frac{1}{12} \times (0.5 \times 300) \times 50^3 + 7500 \times (75 - 41.7)^2$$
$$= 9{,}894{,}166.8 \text{ mm}^4$$

$$I_{bot} = \frac{1}{12} \times 300 \times 50^3 + 15000 \times (41.7 - 25)^2$$
$$= 7{,}293{,}333.5 \text{ mm}^4$$

$$I = 9{,}894{,}166.8 + 7{,}293{,}333.5$$
$$= 17{,}187{,}500$$
$$= 1.719 \times 10^7 \text{ mm}^4$$

Distance to the extreme fibres

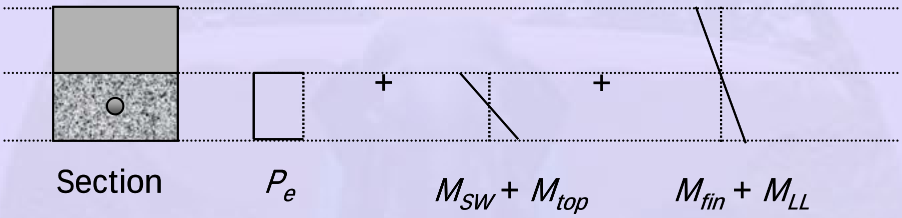
$$y_b = 41.7 \text{ mm}$$
$$y_t = 58.3 \text{ mm}$$

Section moduli

$$Z_b = 17.19 \times 10^6 / 41.7$$
$$= 412{,}527 \text{ mm}^3$$
$$Z_t = 17.19 \times 10^6 / 58.3$$
$$= 294{,}703 \text{ mm}^3$$

3) Calculation of prestress

The tendons are located at the mid depth of the precast plank. Hence, e = 0 for the precast plank. The value of P_e is calculated directly from the following stress profiles.



Stress profiles

To avoid tensile stress at the bottom under service conditions, the resultant stress is equated to zero.

$$-\frac{P_e}{A_1} + \frac{M_{SW} + M_{top}}{Z_{1b}} + \frac{M_{fin} + M_{LL}}{Z_{2b}} = 0$$

$$\text{or,} \quad P_e = A_1 \left[\frac{M_{SW} + M_{top}}{Z_{1b}} + \frac{M_{fin} + M_{LL}}{Z_{2b}} \right]$$

In the above expression, the first term inside the bracket corresponds to the precast section. The moments due to self weight and topping slab are resisted by the precast section alone.

The second term inside the bracket corresponds to the equivalent section. The moments due to weight of the floor finish and live load are resisted by the equivalent section.

$$P = A \left[\frac{0.45 + 0.45}{125{,}000} + \frac{0.54 + 0.72}{412{,}527} \right] \times 10^6$$
$$= 50 \times 300 \times (7.2 + 3.0)$$
$$= 153{,}816 \text{ N}$$

Assuming around 20% loss,

The prestress at transfer (P_0) = 1.2 × 153,816

= 184,579 N.

Wires of diameter = 7 mm and ultimate strength (f_{pk}) = 1500 MPa are selected for prestressing.

Area of one wire (A_p) = 38.48 mm^2.

The maximum allowable tension in one wire

$= 0.8\ f_{pk} \times A_p$

$= 0.8 \times 1500 \times 38.48$

= 46,176 N.

No. wires required = 184,579 / 46,176

= 3.99

→ 4.

Required pull in each wire = 184,579 / 4

= 46,145 N.

Total prestressing force (P_0) = 4 × 46,145

= 184,580 N.

Effective prestressing force (P_e) = 0.8 × 184,580

= 147,664 N.

4) Checking of stresses in concrete

a) At transfer

The compressive strength at 7 days (f_{ci}) $= 0.7\ f_{ck}$

$= 0.7 \times 60$

= 42 MPa.

Allowable compressive stress ($f_{cc,all}$) $= 0.44\ f_{ci}$

$= 0.44 \times 42$

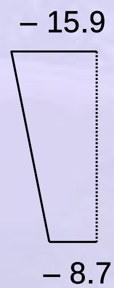
$= 18.5$ MPa.

For Type 1 members, the allowable tensile stress ($f_{ct,all}$) is zero.

Stresses at the mid-span of the precast portion

$f_c = -P_0/A_1 \pm M_{SW}/Z_1$

$= -12.3 \pm (0.45 \times 106 / 125{,}000)$



$f_t = -15.9$ MPa

$f_b = -8.7$ MPa

$\therefore\ |f_t| \leq f_{cc,all}$ OK

b) After casting of topping slab at 28 days

Allowable compressive stress ($f_{cc,all}$) $= 0.44 f_{ck}$

$= 0.44 \times 60$

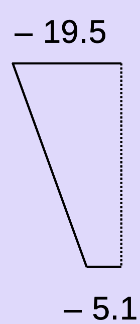
$= 26.4$ MPa.

The allowable tensile stress ($f_{ct,all}$) is zero.

Stresses at the mid-span of the precast portion

$f_c = -P_0/A_1 \pm (M_{SW} + M_{top})/Z_1$

$= -12.31 \pm ((0.45 + 0.45) \times 106 / 125{,}000)$



$f_t = -19.5$ MPa

$f_b = -5.1$ MPa

$\therefore\ |f_t| \leq f_{cc,all}$ OK

c) At service

i) For the precast portion

Allowable compressive stress ($f_{cc,all}$) $= 0.35\, f_{ck}$

$= 0.35 \times 60$

$= 21$ MPa.

The allowable tensile stress ($f_{ct,all}$) is zero.

Stresses at the mid-span of the composite section for unpropped construction

$$f_c = -\,P_e / A_1 \;\pm\; (M_{SW} + M_{top}) / Z_1 \;\pm\; (M_{fin} + M_{LL}) / Z_2$$

$$f_{junc} = -\,(147{,}664 / 15{,}000) - ((0.45 + 0.45) \times 106 / 125{,}000)$$
$$-\,((0.54 + 0.72) \times 106 / 2{,}063{,}625)$$
$$= -\,17.6 \text{ MPa}$$

$$f_b = -\,(147{,}664 / 15{,}000) + ((0.45 + 0.45) \times 106 / 125{,}000)$$
$$+\,((0.54 + 0.72) \times 106 / 412{,}527)$$
$$= 0.4 \text{ MPa}$$
$$\cong 0$$

$|f_{jung}| \leq f_{cc,all}$ OK

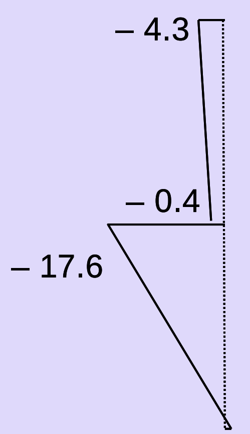
$f_b = f_{ct,all}$ OK

ii) For cast-in-situ portion

Allowable compressive stress ($f_{cc,all}$) $= 0.35\, f_{ck}$

$= 0.35 \times 15$

$= 5.2$ MPa.



Stresses at the mid-span of the composite section

$$f_t = -\,(0.54 + 0.72) \times 106 / 294{,}703$$
$$= -\,4.3 \text{ MPa}$$

$$f_{junc} = -\,(0.54 + 0.72) \times 106 / 2{,}063{,}625)$$

$= -0.6$ MPa

$f_t \leq f_{cc,all}$ OK

Note that the critical stress at the junction is in the precast portion.

5) Check for shear

$V_{uR} = V_c$

$= V_{c0}$

$= 0.67bh\sqrt{(f_t^2 + 0.8f_{cp}f_t)}$

$= 0.67 \times 300 \times 50 \sqrt{(1.86^2 + 0.8 \times 9.36 \times 1.86)}$

$= 41.9$ kN

$V_u = w_uB\ L / 2$

$= 1.5 \times 6 \times 0.3 \times 3.1 / 2 = 4.2$ kN

$V_{uR} \geq V_u$

Therefore, the shear capacity is adequate.

6) Transverse reinforcement

Using Fe 415 grade of steel, for 1m width

$A_{st,min} = 0.12\%\ 1000\ h$

$= 0.0012 \times 1000 \times 100$

$= 120$ mm^2.

Provide 8 mm diameter bars at 300 mm on centre.

7) Provide nominal reinforcement for shrinkage in the longitudinal direction of the topping slab.

Using Fe 415 grade of steel, for 1m width

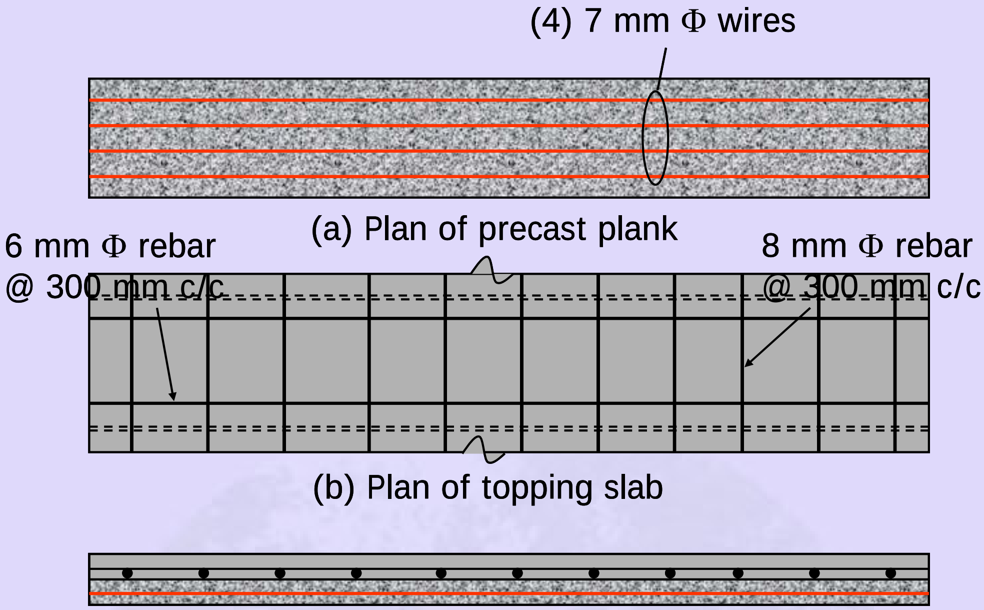
$A_{st,min} = 0.12\%\ 1000\ h$

$= 0.0012 \times 1000 \times 50$

$= 60\ \text{mm}^2$.

Provide 6 mm diameter bars at 300 mm on centre.

Reinforcement details



(a) Plan of precast plank

(b) Plan of topping slab

(c) Longitudinal Section of composite slab

Φ: diameter