Properties of the Z-transform

1. RoC is generally a disk on the z-plane.

$$0 \leq r_R \leq |z| \leq r_L \leq \infty$$

2. Fourier Transform of $x[n]$ converges when RoC includes the unit circle.
3. RoC does not contain any poles.
4. If $x[n]$ is finite duration, RoC contains entire z - plane except for $z = 0$ and $z = \infty$.
5. For a left handed sequence, RoC is bounded by $|z| < min(|a|, |b|)$.
6. For a right handed sequence, RoC is bounded by $|z| > max(|a|; |b|)$.

Inverse Z-transform

To determine the inverse Z-transform, it is necessary to know the RoC.

- RoC decides whether a given signal is causal (exists for positive time), anticausal (exists for negative time) or both causal and anticausal (exists forboth positive and negative time)

Different approaches to compute the inverse Z-transform

- Long division method When Z-Transform is rational, i.e. it can be expressed as the ratio of two polynomials $P(z)$ and $Q(z)$

$$X(z) = \frac{P(z)}{Q(z)}$$

  Then, inverse Z-transform can be obtained using long division:

  – Divide $P(z)$ by $Q(z)$. Let this be:

$$X(z) = \sum_{i=-\infty}^{\infty} a_i z^{-i} \tag{1}$$

- The coefficients of the RHS of equation (1) correspond to the time sequencei.e. the coefficients of the quotient of the long division gives the sequence.

- Partial Fraction method
  - the Z-Transform is decomposed into partial fractions
  - the inverse Z-transform of each fraction is obtained independently
  - the inverse sequences are then added

The method of adding inverse Z-transform is illustrated below. Let,

$$
\begin{aligned}
X(z) &= \frac{\displaystyle\sum_{k=0}^{M} b^k.z^{-k}}{\displaystyle\sum_{k=0}^{N} a^k.z^{-k}}, \qquad\qquad M < N \\
&= \frac{\displaystyle\prod_{k=1}^{M}(1 - c_k.z^{-1})}{\displaystyle\prod_{k=1}^{N}(1 - d_k.z^{-1})} \\
&= \sum_{k=1}^{N} \frac{A_k}{(1 - d_k.z^{-1})}
\end{aligned}
$$

where,

$$A_k = (1 - d_k.z^{-1})X(z)|_{z=d_k}$$

For s multiple poles at $z = d_i$

$$X(z) = \sum_{k=0}^{M-N} B_r.z^{-1} + \sum_{k=1,k\neq i}^{N} \frac{A_k}{(1 - d_k.z^{-1})} + \sum_{m=1}^{s} \frac{C_m}{(1 - d_i.z^{-1})^m}$$